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SHORT COMMUNICATION

Roe linearization for the Euler equations augmented by the convective terms from the k–ω turbulence model

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SUMMARY

This paper describes the method of calculation of the eigenvalues, eigenvectors of the Jacobian matrix of the Euler equations augmented by the convective part of k–ω turbulence model. The equations are 3D and values are expressed in terms of cell normals of a finite volume. The expressions for wave strengths are also found, which are also necessary to calculate the inter-cell fluxes for Roe's scheme. Copyright © 2007 John Wiley & Sons, Ltd.

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KEY WORDS: Roe's scheme; eigenvalues; eigenvectors; wave strengths

1. INTRODUCTION

The cell normal formulation of Euler equations is very convenient from computational point of view in the sense that it allows to update the dependent variables in a single step unlike dimensional splitting. The k–ω turbulence model are frequently used in a low-cost turbulent flow simulation. If the convective part of the k–ω equation could be used together with Euler equations in the calculation of inviscid fluxes, it would ease the process further. In conservative formulation the expression for the augmented Jacobian matrix is so complex that direct calculation of its eigenvectors to use in Roe's scheme [1] is tedious. In the following portion the method of calculation of those quantities is described following the procedure as mentioned in [2]. It is found that judicious choice of arbitrary constants ensures linearly independent set of eigenvectors.

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2. FORMULATION

The Navier–Stokes equations with k–ω turbulence model is given by

$$\frac{\partial}{\partial t}\int_V \mathbf{U}\,\mathrm{d}v+\oint_S(\mathscr{H}-\mathscr{F}).\boldsymbol{\kappa}\,\mathrm{d}s=\int_V \mathbf{S}\,\mathrm{d}v \tag{1}$$

where $\mathbf{U}$ and $\mathbf{S}$ are column vectors containing conservative variables and source terms, respectively. $\mathscr{H}$ and $\mathscr{F}$ are second-order flux-tensors containing inviscid and viscous fluxes, respectively. Here, $\mathbf{U}=[\rho,\ \rho u,\ \rho v,\ \rho w,\ \rho E,\ \rho k,\ \rho\omega]^{\mathrm{T}}$ and $\mathscr{H}=[\rho\mathbf{V},\ \rho u\mathbf{V}+p\mathbf{I}_x,\ \rho v\mathbf{V}+p\mathbf{I}_y,\ \rho w\mathbf{V}+p\mathbf{I}_z,\ (\rho E+p)\mathbf{V},\ \rho k\mathbf{V},\ \rho\omega\mathbf{V}]^{\mathrm{T}}$. The expressions for $\mathbf{S}$ and $\mathscr{F}$ are not given since they are not required in the present context. Here, $\rho,\ u,\ v,\ w,\ p,\ k,\ \omega$ represent density, x,y,z components of velocity, pressure, turbulence kinetic energy and specific dissipation rate, respectively, $\mathbf{V}=u\mathbf{I}_x+v\mathbf{I}_y+w\mathbf{I}_z$, $\mathbf{I}_x,\mathbf{I}_y,\mathbf{I}_z$ are unit vectors along the x,y,z directions, $\boldsymbol{\kappa}$ is the outward normal to ds,

$$E=\frac{p}{\rho(\gamma-1)}+\tfrac{1}{2}(u^2+v^2+w^2)+k$$

3. CALCULATION PROCEDURE

Roe's scheme gives approximate value of $\mathscr{H}_N(U_L,U_R)$ at cell interfaces as

$$\mathscr{H}_N(U_L,U_R)=\frac{1}{2}(\mathscr{H}_N(U_L)+\mathscr{H}_N(U_R))-\frac{1}{2}\sum_{i=1}^{6}\tilde{\alpha}_i|\tilde{\lambda}_i|\tilde{K}^{(i)} \tag{2}$$

where $\mathscr{H}_N(U_L),\mathscr{H}_N(U_R),\tilde{\alpha}_i,\tilde{\lambda}_i,\tilde{K}^{(i)}$ are normal fluxes at left and right states across a cell interface, Roe average wave strengths, eigenvalues and eigenvectors of the matrix $\mathbf{A}.\boldsymbol{\kappa}$, respectively. $\mathbf{A}=\partial\mathscr{H}/\partial\mathbf{U}$ is homogeneous, in other words $\mathscr{H}=\mathbf{A}\mathbf{U}$. As direct calculation of eigenvectors are tedious, an alternative method is followed. In that first the expression for a matrix containing the right eigenvectors of the Jacobian matrix ($\tilde{\mathbf{A}}$) is found for primitive variable formulation of the augmented set of equations and then it is multiplied by the Jacobian matrix of transformation from the conservative to the non-conservative variables, which is defined as $M=\partial\mathbf{U}/\partial\mathbf{V}$ where $\mathbf{V}$ is a column vector containing primitive variables given by $[\rho\ u\ v\ w\ p\ k\ \omega]^{\mathrm{T}}$. The expression for M and $\tilde{\mathbf{A}}$ are given by

$$M=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0\\ u & \rho & 0 & 0 & 0 & 0 & 0\\ v & 0 & \rho & 0 & 0 & 0 & 0\\ w & 0 & 0 & \rho & 0 & 0 & 0\\ \frac{1}{2}V^2+k & \rho u & \rho v & \rho w & \frac{1}{\gamma-1} & \rho & 0\\ k & 0 & 0 & 0 & 0 & \rho & 0\\ \omega & 0 & 0 & 0 & 0 & 0 & \rho \end{bmatrix}$$

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$$\tilde{\mathbf{A}} = \begin{bmatrix} u_\perp & \rho\kappa_x & \rho\kappa_y & \rho\kappa_z & 0 & 0 & 0 \\ 0 & u_\perp & 0 & 0 & \dfrac{\kappa_x}{\rho} & 0 & 0 \\ 0 & 0 & u_\perp & 0 & \dfrac{\kappa_y}{\rho} & 0 & 0 \\ 0 & 0 & 0 & u_\perp & \dfrac{\kappa_z}{\rho} & 0 & 0 \\ 0 & \rho a^2\kappa_x & \rho a^2\kappa_y & \rho a^2\kappa_z & u_\perp & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & u_\perp & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & u_\perp \end{bmatrix}$$

respectively, where $V^2 = \frac{1}{2}(u^2+v^2+w^2)$. The eigenvalues of A are $\lambda = [u_\perp,\ u_\perp,\ u_\perp,\ u_\perp + a\kappa,\ u_\perp - a\kappa,\ u_\perp,\ u_\perp]$, where κ is the modulus of the vector $\boldsymbol{\kappa}$, $u_\perp = u\kappa_x + v\kappa_y + w\kappa_z$, $\kappa_x, \kappa_y, \kappa_z$ are components of $\boldsymbol{\kappa}$, a is speed of sound.

The left eigenvectors l^j are found by solving

$$l^j A = \lambda_j l^j \tag{3}$$

For $\lambda_j = u_\perp$ the following conditions are found for the components of l^j:

$$\begin{aligned} &l_1 \quad \text{arbitrary} \\ &l_2 + a^2 l_5 = 0 \\ &l_3 + a^2 l_5 = 0 \\ &l_4 + a^2 l_5 = 0 \\ &\kappa_x l_2 + \kappa_y l_3 + \kappa_z l_4 = 0 \\ &l_6 \quad \text{arbitrary} \\ &l_7 \quad \text{arbitrary} \end{aligned} \tag{4}$$

The corresponding eigenvectors are

$$\begin{aligned} l^1 &= [\mu_1^1, 0, \kappa_z\mu_2^1, -\kappa_y\mu_2^1, -\mu_1^1/a^2, \mu_3^1, \mu_4^1] \\ l^2 &= [\mu_1^2, 0, \kappa_z\mu_2^2, -\kappa_y\mu_2^2, -\mu_1^2/a^2, \mu_3^2, \mu_4^2] \\ l^3 &= [\mu_1^3, 0, \kappa_z\mu_2^3, -\kappa_y\mu_2^3, -\mu_1^3/a^2, \mu_3^3, \mu_4^3] \\ l^6 &= [\mu_1^6, 0, \kappa_z\mu_2^6, -\kappa_y\mu_2^6, -\mu_1^6/a^2, \mu_3^6, \mu_4^6] \\ l^7 &= [\mu_1^7, 0, \kappa_z\mu_2^7, -\kappa_y\mu_2^7, -\mu_1^7/a^2, \mu_3^7, \mu_4^7] \end{aligned} \tag{5}$$

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For the two remaining eigenvectors, one obtains

$$\begin{aligned} l_1 &= 0 \\ l_2 &= \pm a\hat{\kappa}_x \rho l_5 \\ l_3 &= \pm a\hat{\kappa}_y \rho l_5 \\ l_4 &= \pm a\hat{\kappa}_z \rho l_5 \\ l_5 &\quad \text{arbitrary} \\ l_6 &= 0 \\ l_7 &= 0 \end{aligned} \tag{6}$$

and

$$l^4 = [0, \hat{\kappa}_x \mu^4, \hat{\kappa}_y \mu^4, \hat{\kappa}_z \mu^4, \mu^4/\rho a, 0, 0] \tag{7}$$

$$l^5 = [0, -\hat{\kappa}_x \mu^5, -\hat{\kappa}_y \mu^5, -\hat{\kappa}_z \mu^5, \mu^5/\rho a, 0, 0] \tag{8}$$

where $\hat{\kappa}_x, \hat{\kappa}_y, \hat{\kappa}_z$ are components of unit vectors along $\boldsymbol{\kappa}$. The following values of arbitrary constants (μ) are chosen

$$\begin{aligned} \mu_2^1 = \mu_2^2 = \mu_2^3 &= \frac{1}{\kappa} \\ \mu^4 = \mu^5 &= 1 \\ \mu_1^1 = \hat{\kappa}_x, \quad \mu_1^2 = \hat{\kappa}_y, \quad \mu_1^3 &= \hat{\kappa}_z \\ \mu_3^1 = \mu_4^1 = \mu_3^2 = \mu_4^2 = \mu_3^3 = \mu_4^3 = \mu_4^6 = \mu_3^7 &= 0 \\ \mu_3^6 = \mu_4^7 &= 1 \end{aligned} \tag{9}$$

The matrix (L^{-1}) whose rows contains the left eigenvectors and its inverse are given by

$$L^{-1} = \begin{bmatrix} \hat{\kappa}_x & 0 & \hat{\kappa}_z & -\hat{\kappa}_y & \dfrac{-\hat{\kappa}_x}{a^2} & 0 & 0 \\ \hat{\kappa}_y & -\hat{\kappa}_z & 0 & \hat{\kappa}_x & \dfrac{-\hat{\kappa}_y}{a^2} & 0 & 0 \\ \hat{\kappa}_z & \hat{\kappa}_y & -\hat{\kappa}_x & 0 & \dfrac{-\hat{\kappa}_z}{a^2} & 0 & 0 \\ 0 & \hat{\kappa}_x & \hat{\kappa}_y & \hat{\kappa}_z & \dfrac{1}{\rho a} & 0 & 0 \\ 0 & -\hat{\kappa}_x & -\hat{\kappa}_y & -\hat{\kappa}_z & \dfrac{1}{\rho a} & 0 & 0 \\ \hat{\kappa}_x & 0 & \hat{\kappa}_z & -\hat{\kappa}_y & \dfrac{-\hat{\kappa}_x}{a^2} & 1 & 0 \\ \hat{\kappa}_y & -\hat{\kappa}_z & 0 & \hat{\kappa}_x & \dfrac{-\hat{\kappa}_y}{a^2} & 0 & 1 \end{bmatrix} \tag{10}$$

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$$L=\begin{bmatrix} \hat{\kappa}_x & \hat{\kappa}_y & \hat{\kappa}_z & \dfrac{\rho}{2a} & \dfrac{\rho}{2a} & 0 & 0 \\ 0 & -\hat{\kappa}_z & \hat{\kappa}_y & \dfrac{\hat{\kappa}_x}{2} & -\dfrac{\hat{\kappa}_x}{2} & 0 & 0 \\ \hat{\kappa}_z & 0 & -\hat{\kappa}_x & \dfrac{\hat{\kappa}_y}{2} & -\dfrac{\hat{\kappa}_y}{2} & 0 & 0 \\ -\hat{\kappa}_y & \hat{\kappa}_x & 0 & \dfrac{\hat{\kappa}_z}{2} & -\dfrac{\hat{\kappa}_z}{2} & 0 & 0 \\ 0 & 0 & 0 & \dfrac{\rho a}{2} & \dfrac{\rho a}{2} & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Then L is pre-multiplied by M to obtain the matrix containing right eigenvectors of the Jacobian matrix in conservative formulation. That is given by

$$R=\begin{bmatrix} \hat{\kappa}_x & \hat{\kappa}_y & \hat{\kappa}_z & \dfrac{\rho}{2a} & \dfrac{\rho}{2a} & 0 & 0 \\ u\hat{\kappa}_x & u\hat{\kappa}_y-\rho\hat{\kappa}_z & u\hat{\kappa}_z+\rho\hat{\kappa}_y & \dfrac{\rho u}{2a}+\dfrac{1}{2}\rho\hat{\kappa}_x & \dfrac{\rho u}{2a}-\dfrac{1}{2}\rho\hat{\kappa}_x & 0 & 0 \\ v\hat{\kappa}_x+\rho\hat{\kappa}_z & v\hat{\kappa}_y & v\hat{\kappa}_z-\rho\hat{\kappa}_x & \dfrac{\rho v}{2a}+\dfrac{1}{2}\rho\hat{\kappa}_y & \dfrac{\rho v}{2a}-\dfrac{1}{2}\rho\hat{\kappa}_y & 0 & 0 \\ w\hat{\kappa}_x-\rho\hat{\kappa}_y & w\hat{\kappa}_y+\rho\hat{\kappa}_x & w\hat{\kappa}_z & \dfrac{\rho w}{2a}+\dfrac{1}{2}\rho\hat{\kappa}_z & \dfrac{\rho w}{2a}-\dfrac{1}{2}\rho\hat{\kappa}_z & 0 & 0 \\ -\rho+\mathbf{b}.\mathbf{I}_x & \mathbf{b}.\mathbf{I}_y & \mathbf{b}.\mathbf{I}_z & \dfrac{\rho}{2a}(H+au_\perp) & \dfrac{\rho}{2a}(H-au_\perp) & \rho & 0 \\ -\rho+k\hat{\kappa}_x & k\hat{\kappa}_y & k\hat{\kappa}_z & \dfrac{\rho k}{2a} & \dfrac{\rho k}{2a} & \rho & 0 \\ \omega\hat{\kappa}_x & -\rho+\omega\hat{\kappa}_y & \omega\hat{\kappa}_z & \dfrac{\rho\omega}{2a} & \dfrac{\rho\omega}{2a} & 0 & \rho \end{bmatrix}$$

where $\mathbf{b}=(k+V^2/2)\mathbf{I}_\kappa+\rho(\mathbf{V}\times\mathbf{I}_\kappa)$, $H=a^2/(\gamma-1)+\frac{1}{2}V^2+k$, $\hat{I}_\kappa$ is the unit vector along $\boldsymbol{\kappa}$. The eigenvalues remain the same. The values of wave strengths are calculated by using the relation

$$\Delta\mathbf{U}=\sum_{i=1}^{7}\alpha_i R^i \tag{11}$$

where Δ represents jump across left and right states. The expressions for α's are $\alpha_1=\hat{\kappa}_x(\Delta\rho-\Delta p/a^2)+\hat{\kappa}_z\Delta v-\hat{\kappa}_y\Delta w$, $\alpha_2=\hat{\kappa}_y(\Delta\rho-\Delta p/a^2)+\hat{\kappa}_x\Delta w-\hat{\kappa}_z\Delta u$, $\alpha_3=\hat{\kappa}_z(\Delta\rho-\Delta p/a^2)+\hat{\kappa}_y\Delta u-\hat{\kappa}_x\Delta v$, $\alpha_4=\Delta p/\rho a+\Delta u_\perp$, $\alpha_5=\Delta p/\rho a-\Delta u_\perp$, $\alpha_6=\Delta k+\hat{\kappa}_x(\Delta\rho-\Delta p/a^2)+\hat{\kappa}_z\Delta v-\hat{\kappa}_y\Delta w$, $\alpha_7=\Delta\omega+\hat{\kappa}_y(\Delta\rho-\Delta p/a^2)+\hat{\kappa}_x\Delta w-\hat{\kappa}_z\Delta u$.

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The Roe average states are obtained by solving the equations

$$\Delta\mathbf{U}=\sum_{i=1}^{7}\tilde{\alpha}_i\tilde{R}^i \tag{12}$$

$$\Delta\mathscr{H}_N=\sum_{i=1}^{7}\tilde{\alpha}_i\tilde{\lambda}_i\tilde{R}^i \tag{13}$$

and are given by

$$\begin{aligned}
\tilde{\rho}&=\sqrt{\rho_L\rho_R}\\
\tilde{u}&=\frac{\sqrt{\rho_L}u_L+\sqrt{\rho_R}u_R}{\sqrt{\rho_L}+\sqrt{\rho_R}}\\
\tilde{v}&=\frac{\sqrt{\rho_L}v_L+\sqrt{\rho_R}v_R}{\sqrt{\rho_L}+\sqrt{\rho_R}}\\
\tilde{w}&=\frac{\sqrt{\rho_L}w_L+\sqrt{\rho_R}w_R}{\sqrt{\rho_L}+\sqrt{\rho_R}}\\
\tilde{H}&=\frac{\sqrt{\rho_L}H_L+\sqrt{\rho_R}H_R}{\sqrt{\rho_L}+\sqrt{\rho_R}}\\
\tilde{k}&=\frac{\sqrt{\rho_L}k_L+\sqrt{\rho_R}k_R}{\sqrt{\rho_L}+\sqrt{\rho_R}}\\
\tilde{\omega}&=\frac{\sqrt{\rho_L}\omega_L+\sqrt{\rho_R}\omega_R}{\sqrt{\rho_L}+\sqrt{\rho_R}}
\end{aligned} \tag{14}$$

The primitive variables (ρ,u,v,w,k,ω), a and H in λ_i, α_i and R are replaced by corresponding Roe average values to obtain $\tilde{\lambda}$, $\tilde{\alpha}$ and $\tilde{R}$. $\tilde{a}=[(\gamma-1)(\tilde{H}-\frac{1}{2}\tilde{V}^2-\tilde{k}]^{1/2}$.

4. NUMERICAL TEST CASE

To run a test case the viscous and source terms (due to k–ω equations) need to be used. The expressions for $\mathscr{F}$ and $\mathbf{S}$ of Equation (1) are given by

$$\mathscr{F}=\frac{1}{Re_\infty}\begin{bmatrix}0\\ \tau_{xx}\hat{\kappa}_x+\tau_{yx}\hat{\kappa}_y+\tau_{zx}\hat{\kappa}_z\\ \tau_{xy}\hat{\kappa}_x+\tau_{yy}\hat{\kappa}_y+\tau_{zy}\hat{\kappa}_z\\ \tau_{xz}\hat{\kappa}_x+\tau_{yz}\hat{\kappa}_y+\tau_{zz}\hat{\kappa}_z\\ A\\ (\mu+\sigma^*\mu_t)\dfrac{\partial k}{\partial x}\hat{\kappa}_x+(\mu+\sigma^*\mu_t)\dfrac{\partial k}{\partial y}\hat{\kappa}_y+(\mu+\sigma^*\mu_t)\dfrac{\partial k}{\partial z}\hat{\kappa}_z\\ (\mu+\sigma\mu_t)\dfrac{\partial \omega}{\partial x}\hat{\kappa}_x+(\mu+\sigma\mu_t)\dfrac{\partial \omega}{\partial y}\hat{\kappa}_y+(\mu+\sigma\mu_t)\dfrac{\partial \omega}{\partial z}\hat{\kappa}_z\end{bmatrix}$$

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$$
\mathbf{S}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ S\rho\dfrac{k}{Re_\infty\omega}-\dfrac{2}{3}\rho kD-\beta^* Re_\infty\rho k\omega \\ \alpha\dfrac{\omega}{k}\left(S\rho\dfrac{k}{Re_\infty\omega}-\dfrac{2}{3}\rho kD\right)-\beta Re_\infty\rho\omega^2 \end{bmatrix}
$$

where

$$
\tau_{xx}=\mu_{\text{eff}}\left(\frac{4}{3}\frac{\partial u}{\partial x}-\frac{2}{3}\frac{\partial v}{\partial y}-\frac{2}{3}\frac{\partial w}{\partial z}\right)-\frac{2}{3}\rho k,\quad \tau_{xy}=\mu_{\text{eff}}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),\quad \tau_{xz}=\mu_{\text{eff}}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)
$$

and similarly for other shear stresses, $\mu_{\text{eff}}=\mu+\mu_t$ where μ and μ_t are molecular and eddy viscosities, respectively. μ is determined following Sutherland's law and $\mu_t=\rho k/\omega$, $A=(u\tau_{xx}+v\tau_{yx}+w\tau_{zx}+(\mu+\sigma^*\mu_t)\partial k/\partial x-q_x)\hat{\kappa}_x+(u\tau_{xy}+v\tau_{yy}+w\tau_{zy}+(\mu+\sigma^*\mu_t)\partial k/\partial y-q_y)\hat{\kappa}_y+(u\tau_{xz}+v\tau_{yz}+w\tau_{zz}+(\mu+\sigma^*\mu_t)\partial k/\partial z-q_z)\hat{\kappa}_z$

$$
q_x=-\left(\frac{\mu}{Pr(\gamma-1)}+\frac{\mu_t}{Pr_t(\gamma-1)}\right)\frac{\partial a^2}{\partial x}
$$

q_y, q_z follow similarly, here $Pr=0.71$ and $Pr_t=0.9$ are moleculer and turbulent Prandtl number respectively, $\sigma=\sigma^*=\frac{1}{2}$,

$$
\begin{aligned}
S=&\left(\frac{4}{3}\frac{\partial u}{\partial x}-\frac{2}{3}\frac{\partial v}{\partial y}-\frac{2}{3}\frac{\partial w}{\partial z}\right)\frac{\partial u}{\partial x}+\left(\frac{4}{3}\frac{\partial v}{\partial y}-\frac{2}{3}\frac{\partial u}{\partial x}-\frac{2}{3}\frac{\partial w}{\partial z}\right)\frac{\partial v}{\partial y}+\left(\frac{4}{3}\frac{\partial w}{\partial z}-\frac{2}{3}\frac{\partial u}{\partial x}-\frac{2}{3}\frac{\partial v}{\partial y}\right)\frac{\partial w}{\partial z}\\
&+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)^2,\quad D=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}
\end{aligned}
$$

$\beta=\frac{3}{40}$, $\beta^*=\frac{9}{100}$, $\alpha=\frac{5}{9}$.

The inviscid and viscous terms of Equation (1) are integrated in time explicitly based on a multi-stage Runge–Kutta method [3], while the source terms are integrated implicitly following the method given in [4]. Viscous fluxes are discretized by central differencing as given in [5]. To increase the spatial accuracy to third order, a MUSCL extrapolation strategy [6] is followed with Van Albada limiter [7] to suppress spurious oscillations near a strong discontinuity.

The numerical results are compared with experimental data points of supersonic turbulent boundary layer velocity profile over a flat plate as reported in [8, 9]. Although a 2D simulation is sufficient for the test case, a 3D simulation is carried out because the present formulation is for 3D case. The Reynolds number (Re_∞) and Mach number are 4.5×10^6 and 4.5, respectively. A total of $121\times61\times51$ grids points are used. Boundary conditions are same as in [8]. In spanwise direction extrapolation is used. Figure 1 shows the variation of normalized velocity profile $u^+=u/u_\tau$ against wall normal co-ordinate $y^+=yu_\tau/v$. Here, u_τ is friction velocity.

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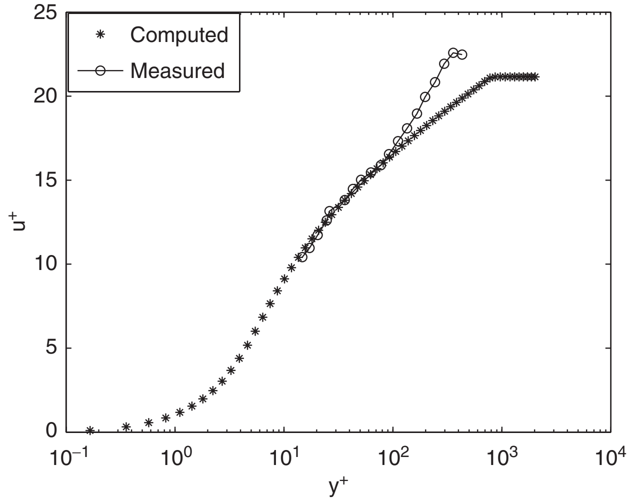

Figure 1. Comparison of computed and measured [9] velocity profiles.

5. CONCLUSION

The eigenvalues, eigenvectors and wave strengths of Roe's scheme are calculated for the Euler equations augmented by convective part of the k–ω turbulence model. The arbitrary constants while determining the eigenvectors are judiciously chosen so that the set is linearly independent and components of individual vectors are simple in form. A 3D test case is considered to check the formulation.

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